DETERMINATION of the dynamic viscosity coefficient of the Stokes viscometer – construction of a measuring set in the Physical Laboratory of the State Higher Vocational School in Tarnów

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Abstract
A Stokes viscometer made and launched by the authors in the Physical Laboratory of the State Higher Vocational School in Tarnów was presented. The construction of the viscometer is discussed and the theoretical description of the physical phenomena occurring there is given. The results of measurements on the device, statistical analysis of the measurement uncertainties and confrontation with the literature value are presented - the obtained results correspond very well to the literature values. A possible further development in the accuracy of theoretical description and experimental measurements related to the ellipsoid analysis of the shape of water droplets was suggested. The didactic aspects of the new experiment in the context of the understanding of molecular physics, especially by students of material engineering, were discussed.

Keywords: Viscosity, Stokes' Law, density measurements, didactic experiment

Introduction

Viscosity is one of the principal parameters describing physicochemical properties of liquids. Familiarization with this issue by students of technical faculties, especially Material Engineering, is crucial in their education process. As a consequence of this fact, the decision to build a Stokes viscometer has been made. The viscometer should allow to examine the aforementioned properties of liquids during lectures in physics laboratory. The choice of the classic viscous method was not accidental. The Stokes method combines simplicity with deep physical content and is great for exploration in the Physics Laboratory (measurements of time, volume, distance, temperature). A properly sensitive viscometer requires the control of thermodynamic parameters of liquids, therefore students are thought not only molecular physics, but also thermodynamics.

The essence of measuring liquid viscosity using Stokes' viscometer

The frictional forces arise during the flow of all real liquids. This property is called viscosity (internal friction). The idea of viscosity is that the force F acts between two flat elements of parallel layers of liquid, that area is S and the distance between them is l. Layers move at low velocities different by Δυ. The proportionality coefficient η between velocity gradient and the pressure was introduced by Newton and is called the viscosity.

In contrast to the motion of solids, in which the friction occurs only on the surface, in liquids and gases, friction appears in the entire volume. It is called internal friction or viscosity.

In order to introduce the concept of viscosity let assume that there is a portion of liquid between two flat plates with a S surface, as shown in Figure 1. If one of the plates moves relative to the other with a small speed v, then the force necessary to maintain the movement is proportional to the surface S and velocity v^2, and inversely proportional to the distance d between the plates. The constant η is called the viscosity coefficient. The unit of η in the SI system is [Pa · s]. Another name for unit is puaz (=0.1 $Pa \cdot s$).

$$\frac{\vec{F}}{S} = \eta \frac{\vec{v}}{d} \tag{1}$$

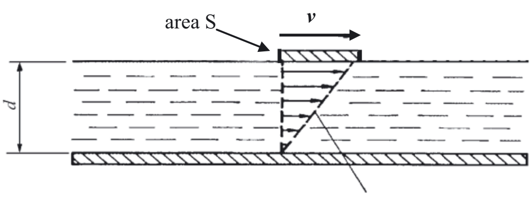

Figure 1. Distribution of fluid velocity in volume between two parallel plates with motion of the tope one

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The viscosity coefficient depends as well on the type of liquid as on the temperature. However, the influence of the temperature on the changes of viscosity coefficient value is crucial. In liquids, the mutual shifting of adjacent layers counteracts cohesion forces and such movements are possible mainly due to the mobility of molecules penetrating from one layer to another. The exchange of particles between the layers, which increases with the increase of temperature, causes the viscosity to decrease as the temperature increases [1, 2].

For gases, the viscosity increases in proportion to the absolute temperature. Whereas, for liquids the viscosity decreases significantly with increasing temperature. A very strong temperature dependence is observed for liquids with high viscosity value, for example for glycerine or motor oils.

When a solid body moves relative to a fluid, viscous drag have to be present. Considering the case that the metal sphere is moving, it carries a layer of fluid adhering to it, and also causes the next layers to move as a result of friction inside the fluid itself (Fig.2). The magnitude of frictional force of the liquid exerted on the moving sphere is proportional to its velocity. It is expressed by the Stokes' formula:

$$F_T = -6\pi\eta r v \quad (2)$$

Where r – the radius of the sphere, v – the velocity of the moving sphere, η – the coefficient of viscosity of the liquid.

The above described formula is known in the literature under the name Stokes' law [1, 2, 3].

Formula (2) is valid when the sphere is moving in an infinite volume of liquid and the velocity of the sphere is much lower that the speed of sound in a given medium. In case when the ball moves along the axis of the cylinder of radius R the formula (2) takes the form [1, 4]:

$$F_T = -6\pi\eta r v \left(1 + 2.4\frac{r}{R}\right) \quad (3)$$

Instead of metal sphere one can use drops of distilled water. It is also possible to construct the Stokes' viscometer using vertical cylinder and a buret with destilld water.

If the ball made of material with a density under the influence of gravity falls in the fluid with a density smaller than the density of the ball, three forces act on it:

gravity:

$$F_G = m_K g = \frac{4}{3}\pi r^3 \rho_K g \quad (4)$$

The buoyant force, which in accordance with Archimedes' principle is:

$$F_W = -m_P g = -\frac{4}{3}\pi r^3 \rho_P g \quad (5)$$

The resistance force is expressed by the Stokes' formula (3).

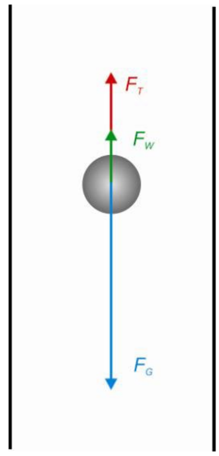

Figure 2. Visualisation of forces acting on water drop moving in the measuring cylinder

The ball, falling in the cylinder, initially moves in a variable motion. Since the Stokes' drag force depends on the speed, increasing with this speed, then there is a limiting value of the ball's speed at which the net force acting on the ball is equal to zero. This means that starting from this time onward, the ball moves further in a uniform motion [5].

According to the Newton's second low of motion, the equation of motion for the ball is:

$$m_K\vec{a} = \vec{F_T} + \vec{F_G} + \vec{F_W} \quad (6)$$

If at the initial moment $t = 0$ the speed $v = v_0$, then the solution of the motion equation (6) leads to the dependence of the speed on time in the form:

$$v(t) = v_{gr} + \left(v_0 - v_{gr}\right) \cdot e^{\frac{t}{\tau}} \quad (7)$$

where the τ is a time constant and v_{gr} is the terminal velocity of the ball.

The speed-time relationship for a ball moving in a viscous liquid is shown in a figure 3. The exponential dependence of the ball's speed (solid line) tends to the terminal value v_{gr} what is clearly visible in comparison to a straight line of constant acceleration motion (broken line) in the figure.

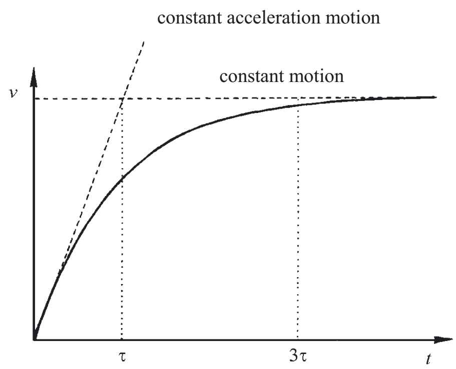

Figure 3. Velocity of a water drop falling with fluid resistance as a function of time. Achievement of a constant motion with the terminal velocity of the drop and v_{gr} is shown

As a result, the movement of the ball after a time of about 3τ becomes practically uniform at a limiting speed equal to:

$$v_{gr} = \frac{(m_K - \rho_P V_P)\cdot g}{6\pi\eta r\left(1+2.4\frac{r}{R}\right)} \tag{8}$$

where V_p is the volume of liquid displaced by the ball.

Therefore, the terminal speed measurement should be made on the section of the path on which the ball has already reached the constant speed. Then from formula (8) we get:

$$\eta = \frac{(m_K - \rho_P V_P)\cdot g}{6\pi v_{gr} r\left(1+2.4\frac{r}{R}\right)} \tag{9}$$

When the ball is a drop of water, then

$$m_K = \frac{4}{3}\pi r^3 \cdot \rho_W \tag{10}$$

where ρ_W is the density of distilled water.

And

$$V_P = \frac{4}{3}\pi r^3 \tag{11}$$

The formulas (10) and (11) allow the formula (9) to be modified to the final form:

$$\eta = \frac{2(\rho_W - \rho_P)\cdot g r^2 t}{9l\left(1+2.4\frac{r}{R}\right)} \tag{12}$$

where $v_{gr} = \frac{l}{t}$, and t is the passage time of the water drop over a distance l, t and l are measured directly along the cylinder, in the area where the movement is with constant velocity [3, 6].

In order to determine the viscosity, all the values appearing in formula (11), excluding the water density and standard acceleration due to gravity, should be measured. The strong dependence of the density of paraffin oil on the temperature requires the measurement of this density based on the U-shaped manometer.

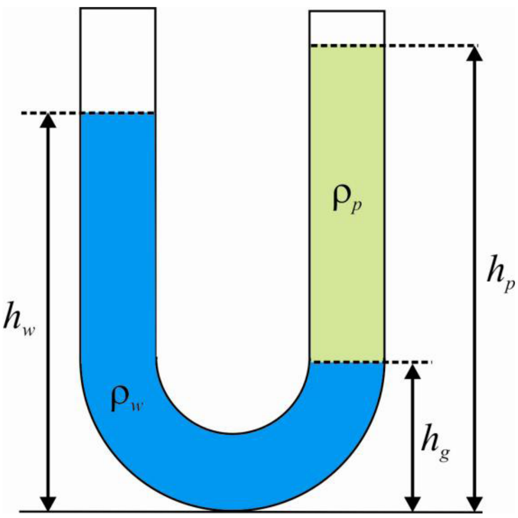

Figure 4. Scheme of a U-shaped manometer containing two types of liquids. The measurement of density ρ_P can be done by knowing the liquid levels h_p, h_W, h_g and density ρ_W (control liquid)

Communicating vessels allow to determine the unknown density of the liquid contained in one vessel and express it in terms of the known density of liquid in the second vessel. When one fills the U-shaped arms with different liquids, the liquid levels in the tube arms are set at different heights. If there are distilled water with density and oil of unknown density in the vessels, then the equality of pressures requires that the following relationship [3]:

$$\rho_P = \rho_W \cdot \frac{h_W - h_g}{h_P - h_g} \tag{13}$$

where h_p is the oil level, h_W is the water level and h_g is the level of the interface separating the two liquids.

Measurement method

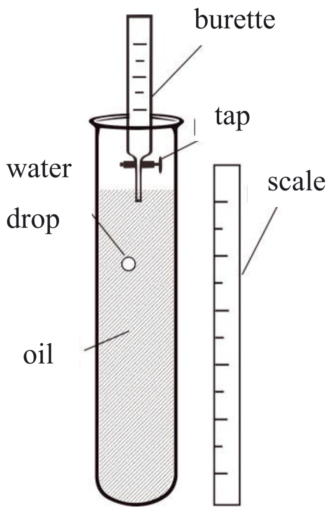

Figure 5. Scheme of the Stokes' viscometer

The experimental Stokes' viscometer system consists of a glass cylinder filled with paraffin oil, fixed in a vertical position, and a linear 10 cm scale. The movable burette with a Teflon tap attached to the top of the glass tube allows the application of distilled water droplets to the cylinder volume. Droplet size can be adjusted by changing the opening of the tap and its replaceable tips. The U-tube manometer is also available at the experimental table, to the arms of which oil and distilled water are used. The distilled water is used in the Stokes' viscometer. There are also enclosed physical tables of density of distilled water as a function of temperature (with the quantized temperature steps of 1°C).

In addition, the following measuring instruments are necessary: stopwatch, thermometer and caliper.

Conducting the experiment requires a series of actions [7]:

- Measurement of air temperature;
- Reading from the physics' table the density of distilled water at a given temperature;

- Reading the two upper and one lower levels of the tested liquids in the U-shape;
- Releasing a drop of distilled water in the cylinder axis and adjusting the burette's operation - forming a stream of droplets convenient for measuring;
- Reading the set water level in the burette;
- Measurement (several times) of the passage time of selected balls over a fixed distance, while counting the total number of drops.
- Closing the burette tap and re-reading the set water level in the burette.

NOTE: the actions described above require the coordination of three people. At the same time one of the experimenters has to count the drops, the second one has to measure the time of flight of selected drops, and the third has to handle the burette.

- Also, very important is to carry out following steps:
- Measurement of the internal diameter of the cylinder;
- Record the accuracy class of the used instruments and measurement uncertainties.

It is important to emphasize that in fact the drop of water falling in the oil environment takes the shape of an ellipsoid. This is indeed a consequence of the anisotropy of the resistance forces acting during the uniform motion. Thus, the most appropriate model to calculate the frontal resistance force would be the resistance model developed already in Rayleight's works at the end of the 19th century. For slowly moving any triaxial ellipsoid with axes a, b, c (laminar flow – a small Reynolds number Re) friction calculations based on the solution of the Stokes' equation where carried out e.g. in [8] and lead to the result:

$$F_T = -6\pi\eta Y v \tag{14}$$

where $Y = \frac{8}{3}\cdot\frac{a\cdot b\cdot c}{X_0+\alpha_0 a^2}$, X_0 and α_0 – parameters resulting from the boundary conditions of the flow. It was assumed that the velocity vector coincides with the semi-axis a of the ellipsoid. One can easily reduce the above formula, in a specific cases $Y = r$ (sphere $a = b = c = r$), $Y = \frac{8}{3\pi}\cdot r$ (disk with radius r moving parallel to the axis of symmetry), $Y = \frac{16}{9\pi}\cdot r$ (disk with radius r moving perpendicular to the axis of symmetry), to the commonly known equations. It is worth to add that the measurement of the half-axis of the ellipsoidal water drop, even in the case of photographs of this drop, will be seriously hampered by optical focusing by the measuring cylinder.

Examples of experimental results

The sample results of measurements carried out on our set are presented.

The ambient temperature measurement was made using a K-type thermocouple connected to a digital multimeter. The measured temperature is $T_{environment} = 23.0°C$ with measurement uncertainty of $\Delta T_{environment} = 0.5°C$. The density of distilled water (from the tables) at $23.0°C$ is $\rho_W = 997.56\frac{kg}{m^3}$. Next, the heights of the water level $h_W = 7.5\ cm$, the oil level $h_P = 8.1\ cm$ and the level of the liquid boundary $h_g = 2.0\ cm$ were read from the U-shaped manometer. Next, from the formula (13), one calculates the density of paraffin oil, obtaining $\rho_P = 899.44\frac{kg}{m^3}$. The measurement uncertainty was calculated as a sum of absolute values of component uncertainties [3]:

$$\Delta\rho_P = \left|\frac{\partial\rho_P}{\partial h_W}\cdot\Delta h_W\right| + \left|\frac{\partial\rho_P}{\partial h_P}\cdot\Delta h_P\right| + \left|\frac{\partial\rho_P}{\partial h_g}\cdot\Delta h_g\right| \tag{15}$$

where $\Delta h_W = \Delta h_P = \Delta h_g \equiv \Delta x = 1\ mm$ is uncertainty of read from U-shape tube arms.

The measurement uncertainty given by the formula (15) can be expressed as:

$$\Delta\rho_P = \frac{2\rho_W}{h_P - h_g}\cdot\Delta x = 32{,}71\frac{kg}{m^3} \tag{16}$$

For further calculations the following value of the density of paraffin oil is taken: $\rho_P = 899(33)\frac{kg}{m^3}$.

Three measuring series were carried out for various burette endings with droppers with variable cross-sections. In each of these series, the time of flight of a drop of distilled water over a specified distance was measured. In addition, the total number of droplets flowing out from the burette was counted. Before and after the measurements, distilled water levels in the burette were read. Each of the measurement series was repeated three times. Example experimental results for measurements carried out for the largest cross-section of the dropper tip are shown in Table 1.

Table 1. Three series of measurements of the time of flight (t) of a drop of distilled water in paraffin oil at $23.0°C$ over a distance of 0.2 m for the largest cross-section of the dropper.

No.	t [s] (Series 1)	t [s] (Series 2)	t [s] (Series 3)
1	7.94	7.21	7.71
2	7.70	8.06	7.87
3	7.80	8.05	7.55
4	7.70	7.68	7.85
5	7.55	7.90	7.77
6	8.16	8.79	7.95
7	7.69	7.81	7.86
8	8.03	7.74	8.30
Mean value for all droplets	7.82	7.91	7.86
n [a]	18	11	12
V [cm^3] [b]	5.8	3.4	3.7

[a] The total amount of droplets released in a given series.
[b] The volume of distilled water used to release all the droplets in a given series.

On the basis of the data in the Table 1, the average drop radius (r) was determined for each measurement series. Assuming that the drop is a ball, one can find: $r = \sqrt[3]{\frac{3V}{4\pi \cdot n}}$. Next, to determine the measurement uncertainty, the following formula was used: $\Delta r = \left|\frac{\partial r}{\partial V} \cdot \Delta V\right| = \sqrt[3]{\frac{1}{36\pi \cdot V^2 \cdot n}}$. Substituting the given data, we find the average drop radius $r = 0.4219(22)\ cm$.

The mean time of flight of a drop over the chosen distance $l = 20\ cm$ is $\Delta t = 7.861(61)\ s$. The measurement uncertainty was calculated as the standard deviation of the mean of all measurements.

Knowledge of the values of the above-determined physical quantities (paraffin oil density ρ_P, ball radius r, time of flight t, distance l passed by the balls, internal radius R of the cylinder) and common knowledge of the gravitational acceleration values and water density allows to determine oil viscosity η from the formula (12). The measurement uncertainty [3] of η can be determined as a sum of absolute values of component uncertainties: $\Delta\eta = \left|\frac{\partial \eta}{\partial \rho_P} \cdot \Delta\rho_P\right| + \left|\frac{\partial \eta}{\partial r} \cdot \Delta r\right| + \left|\frac{\partial \eta}{\partial t} \cdot \Delta t\right| + \left|\frac{\partial \eta}{\partial l} \cdot \Delta l\right| + \left|\frac{\partial \eta}{\partial R} \cdot \Delta R\right| = \eta \cdot \left(\frac{\Delta\rho_P}{\rho_W - \rho_P} + \frac{2+2.4\cdot\frac{r}{R}}{1+2.4\cdot\frac{r}{R}} \cdot \frac{r}{R} + \frac{\Delta t}{t} + \frac{\Delta l}{l} + \frac{2.4\cdot\frac{r}{R^2}}{1+2.4\cdot\frac{r}{R}} \cdot \Delta R\right)$.

Measurement uncertainty *sl* of the distance *l* on which the time of flight of the drop was measured was taken as 1 *cm*. This is an approximate value, estimated from the dynamics of the system. The uncertainty of measuring the internal diameter of the cylinder ΔR is 0.1 *mm*, which is related to the measuring accuracy of the used caliper.

The obtained value of the viscosity of the paraffin oil for the discussed first series of measurements is $\eta_1 = 0.108(44)\ Pa \cdot s$.

The viscosity values of paraffin oil for the measurements series for the intermediate and smallest cross-section of the dropper tip were obtained analogously. The obtained values are: $\eta_2 = 0.084(35)\ Pa \cdot s$ and $\eta_3 = 0.104(43)\ Pa \cdot s$.

Based on calculations for three different dropper tips, the viscosity of the tested paraffin oil was determined: $\eta = 0.099(41)\ Pa \cdot s$. This value was taken as the average of the measurements for the various dropper tips. The paraffin oil viscosity is a physical parameter that strongly depends on the chemical and stochiometric composition. Thus, in literature data the value of viscosity parameter is always given as a range. For instance, producer of used paraffin oil gives the range of viscosity value as 0.110 – 0.230 $Pa \cdot s$.

The role of the discussed experiment in the didactic process

Measuring the viscosity of liquid using the Stokes method allow students to be familiarized with measurements of time, distance, volume, and temperature in molecular physics simultaneously. Typically, these quantities are associated with experience in the field of mechanics and thermodynamics [3]. Such experience is necessary for students of technical faculties of universities on their subsequent years of education and beyond the frame works drawn up by standard teaching modules.

The nature of the experiment requires a multitude of measurements (a large number of drops in the burette, a dozen of time of flights or so measurement). This involves the need for a professional, statistical analysis of these results. The experimenter must know the basic tools of statistical analysis - such as mean value, standard deviation, confidence interval, and also correctly calculate indirect uncertainties. What is more, the ability to correctly prepare graphs including marked bars specifying measurement uncertainties is also required.

Regarding the peculiarity of the experiment, the density of used distilled water must be controlled with an accuracy exceeding the usual standard specification of this parameter. Therefore, students have at their disposal proper tables of the temperature dependence of the density of distilled water.

Another important parameter needed is the density of the paraffin oil. Its average value given by the manufacturer is insufficient. For this reason, students on the auxiliary hydrostatic set (U-shaped) measure the current oil density for a given temperature. They acquire the skills of hydrostatic measurements.

During the experiment it is advisable to sensitize the student to the secondary effects occurring here (the influence of the edge of the vessel, the ellipsoidity of water drops, the effect of parallax, meniscus, distortion of the image through the cylindrical shape of the measuring tube). The peculiarity of the experiment is the abundant occurrence of the above-mentioned secondary effects and the practical knowledge of the need to verify them when constructing the experimental model.

Operation of the Stokes viscometer is not possible for one person; it is necessary to control the number of droplets flowing from the burette and the kinematic parameters of their movement (distance, time), at the same time. Observation window here is spatially differentiated and it is advisable that two or even three people participate in the measurement. This requires coordination of the group's activities and must be pre-exercised as part of the trial measurements. A clear and concise communication system is also required.

Conducting this experiment allows to teach in a non-standard and consistent way such important aspects as:

– Application of time, distance and temperature measurements to issues of molecular physics
– Cooperation in a group – the need to divide roles into to hadle correctly measurement process
– Developing the ability to analyze measurement uncertainties
– Raising awareness of students to the accuracy of reading physicochemical parameters of water.
– Acquiring basic elementary hydrostatic measurements.
– Ability to analyze the influence of secondary effects on the final result of the experiment.

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